RESERVE BANK OF AUSTRALIA

Fiscal Dominance *Risk* According to HANK

Tommy Iao*

Reserve Bank of Australia

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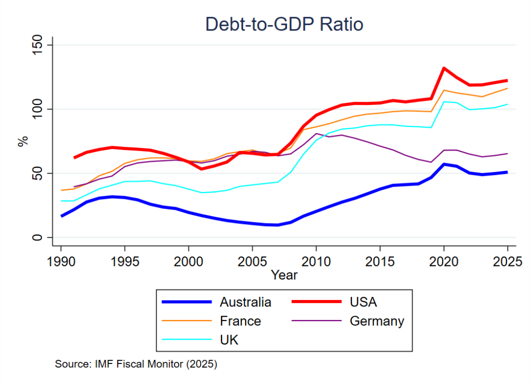
**This work is part of my dissertation completed prior to joining the RBA. Views expressed are my own and do not necessarily reflect the views of the RBA.*

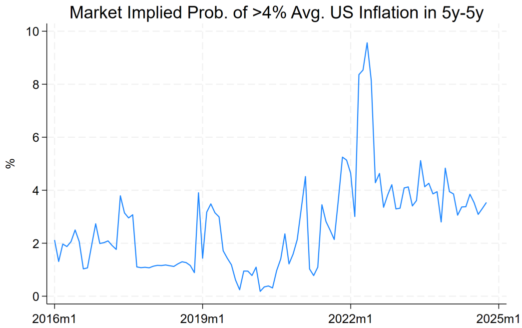
Motivation

- Covid-era fiscal stimulus sharply increased public debt in advanced economies
- Historically high public debt has sparked concerns about fiscal dominance
  - **Fiscal Dominance.** CB serves fiscal needs by lowering interest rates low

Rising Public Debt and Inflation Risk



(a) Debt-to-GDP across adv. economies



(b) Inflation disaster risk (Hilscher et al., 2025)

Motivation

- Covid-era fiscal stimulus sharply increased public debt in advanced economies
- Historically high public debt has sparked concerns about fiscal dominance
  - **Fiscal Dominance.** CB serves fiscal needs by lowering interest rates low
- HANK prominent for analyzing fiscal stimulus, but not fiscal dominance risk
  - Existing work exclusively in RANK, e.g. Bigio et al. (2025); Bianchi and Melosi (2022); Davig and Leeper (2007), etc.

This Paper

- HANK framework with monetary-fiscal regime uncertainty
  - **New channel.** endogenous movement of the natural rate of interest r^*
  - Non-Ricardian behaviors amplify the transmission of fiscal dominance risk
- Apply the framework to interpret the post-pandemic rise in long-term interest rates
  - **Result.** Covid stimulus accounts for 0.9pp$\uparrow$ in 10-year treasury yield and 0.3pp.$\uparrow$ in r^*
- Optimal monetary policy under fiscal dominance risk
  - **Finding.** optimal MP accommodates deficit-financed stimulus, regardless of HA

Today

1. Analytical framework: RANK and BiU
    - inspect the mechanisms and the role of non-Ricardian households

2. Quantitative HANK with fiscal dominance risk
    - application: long-term effect of US Covid stimulus

3. Optimal monetary policy

4. Conclusion

Analytical framework: RANK and BiU

Textbook RANK Model

Two policy regimes (Leeper, 1991)

$$
\begin{aligned}
y_t &= -(r_t^n - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t y_{t+1} \\
\pi_t &= \kappa y_t + \beta \mathbb{E}_t \pi_{t+1} \\
r_t^n &= \phi_\pi \pi_t \\
b_{t+1} &= r_t^n + \rho_b R (b_t - \pi_t)
\end{aligned}
$$

Textbook RANK Model

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$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \pi_{t+1}$$

$$r_t^n = \underbrace{\phi_\pi}_{>1} \pi_t$$

$$b_{t+1} = r_t^n + \underbrace{\rho_b R}_{<1} (b_t - \pi_t)$$

Two policy regimes (Leeper, 1991)

1. Monetary-led (ML) regime:
   - $\phi_\pi > 1$ and $\rho_b R < 1$
   - Gov't debt dynamics does not matter
   - Ricardian Equivalence

Textbook RANK Model

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Two policy regimes (Leeper, 1991)

1. Monetary-led (ML) regime:
   - $\phi_\pi > 1$ and $\rho_b R < 1$
   - Gov't debt dynamics does not matter
   - Ricardian Equivalence

2. Fiscal-dominant (FD) regime:
   - $\phi_\pi < 1$ and $\rho_b R > 1$
   - Inflation adjusts to stabilize debt
   - Fiscal Theory of the Price Level

Fiscal Dominance in RANK

- Consider the FD regime with $\phi_\pi = 0$ and $\rho_b = 1$. Then

$$b_{t+1} = R(b_t - \pi_t)$$

- PDV of inflation ($\approx$ long-run price level) pin down by initial debt:

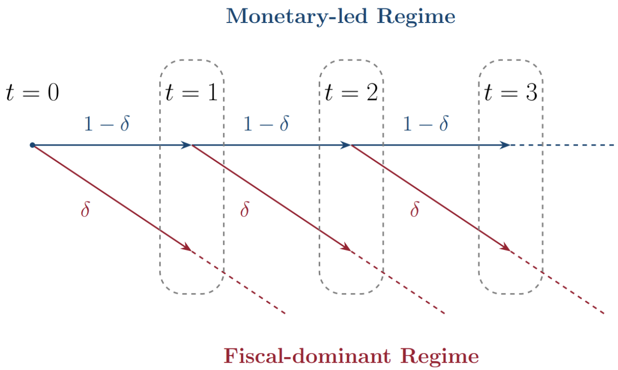
$$\sum_{j=0}^{\infty} R^{-j} \pi_{t+j} = b_t$$

- Output and inflation are thus function of outstanding debt

$$\exists \psi_y, \psi_\pi > 0 : \quad y_t = \psi_y b_t, \quad \pi_t = \psi_\pi b_t$$

Modeling Fiscal Dominance Risk

- Economy begins in the ML regime
- Stochastically transition to the FD regime w.p. $\delta \in (0,1)$ every period
  - Assume that FD regime is absorbing $\Rightarrow$ tractability



RANK with Fiscal Dominance Risk

Along the ML-regime path:

$$
\begin{aligned}
y_t &= -\{r_t^n - [(1-\delta)\pi_{t+1} + \delta\pi_{t+1}^F]\} + (1-\delta)y_{t+1} + \delta y_{t+1}^F \\
\pi_t &= \kappa y_t + \beta[(1-\delta)\pi_{t+1} + \delta\pi_{t+1}^F] \\
r_t^n &= \phi_\pi \pi_t \\
b_{t+1} &= r_t^n + \rho_b R(b_t - \pi_t)
\end{aligned}
$$

- Expectation takes into account possible fiscal dominance in the future

RANK with Fiscal Dominance Risk

Along the ML-regime path:

$$
\begin{aligned}
y_t &= -\left\{r_t^n - [(1-\delta)\pi_{t+1} + \delta\psi_\pi b_{t+1}]\right\} + (1-\delta)y_{t+1} + \delta\psi_y b_{t+1} \\
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- Expectation takes into account possible fiscal dominance in the future
- Debt dynamics matters even under the ML regime

RANK with Fiscal Dominance Risk

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y_t &= -\left\{r_t^n - \left[(1-\delta)\pi_{t+1} + \delta\psi_\pi b_{t+1}\right]\right\} + (1-\delta)y_{t+1} + \delta\psi_y b_{t+1} \\
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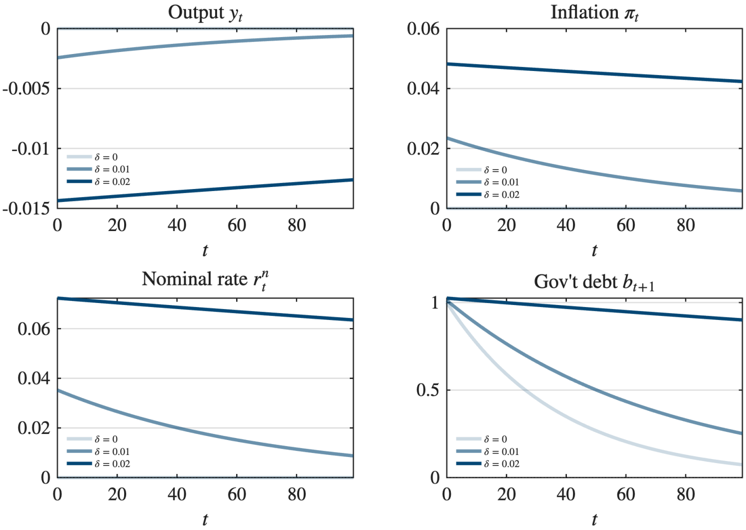
b_{t+1} &= r_t^n + \rho_b R(b_t - \pi_t)
\end{aligned}
$$

- Expectation takes into account possible fiscal dominance in the future
- Debt dynamics matters even under the ML regime
- **Debt-inflation spiral**: $b\uparrow \rightsquigarrow \mathbb{E}\pi\uparrow \rightsquigarrow \pi\uparrow \rightsquigarrow r^n\uparrow \rightsquigarrow b\uparrow$ (Bigio et al., 2025)
  - "Monetary policy stepping on a rake" (Sims, 2011)

Deficit shock IRF



Bond-in-Utility Model

- Introduce non-Ricardian behaviors by assuming bond-in-utility (BiU) preference:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t + \varphi^b \frac{(B_t/\Pi_t)^{1-\eta}}{1-\eta} \right]$$

  BiU captures precautionary saving motive or convenience yield.

- Log-linearized Euler equation:

$$y_t = -\left[r_t^n - \mathbb{E}_t \pi_{t+1} - (1-\alpha)\eta(b_{t+1} - \mathbb{E}_t \pi_{t+1}) \right] + \alpha \mathbb{E}_t y_{t+1}$$

  where $\alpha = \beta R < 1$.

- If $\eta > 0$, gov't debt b_{t+1} shifts aggregate demand $\Rightarrow$ η measures non-Ricardianess.

Fiscal Dominance in non-Ricardian Economies

Proposition (Fiscal dominance is more inflationary)

Suppose $\alpha \in (0,1)$ *and* $\eta > 0$ *satisfy* $\eta(1-\alpha) < 1$. *There exists a unique bounded equilibrium where* $y_t = \psi_y b_t$ *and* $\pi_t = \psi_\pi b_t$ *with* $\psi_y, \psi_\pi > 0$. *Moreover, we have*

$$\frac{\partial \psi_y}{\partial \eta} > 0 \quad \textit{and} \quad \frac{\partial \psi_\pi}{\partial \eta} > 0.$$

Intuition. High MPC households spend down excess savings, boosting aggregate demand.

- The debt-inflation spiral is **amplified** in non-Ricardian economies.

The Natural Rate of Interest r^*

- Define the **natural rate of interest** r^* as the ex-ante real rate that is consistent with $y_t = 0$ in the ML regime
  - What CB should target to close the output gap; consistent with empirical notions of r^*

- Consider the case of no FD risk ($\delta = 0$). The Euler equation implies

$$r_t^* = (1-\alpha)\eta(b_{t+1} - \mathbb{E}_t \pi_{t+1})$$

  Upward-sloping aggregate saving supply $\Rightarrow$ **higher** b_{t+1}**, higher** r_t^*.

- Government debt equation

$$b_{t+1} = \underbrace{r_t^* + \pi_{t+1}}_{=r_t^n} + \rho_B R(b_t - \pi_t)$$

  r^* interacts with debt-inflation spiral: $b \uparrow \rightsquigarrow \mathbb{E}\pi \uparrow \,\&\, r_t^* \uparrow \rightsquigarrow r^n \uparrow \rightsquigarrow b \uparrow$.

Fiscal Dominance Risk and r^*

Proposition

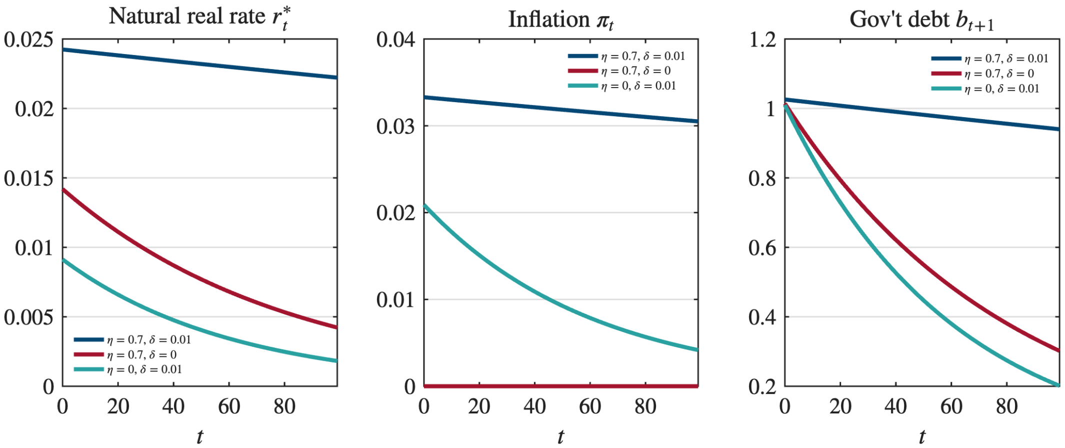
Suppose $\alpha, \eta \in (0,1)$ *satisfy* $(1-\alpha)\eta < 1-\rho_B$. *There exists* $\bar{\delta} > 0$ *such that if* $\delta \in [0, \bar{\delta})$, *the followings are true:*

1. *There exists a unique bounded equilibrium where* $r_t^* = \psi_r \hat{b}_t$ *and* $\hat{b}_{t+1} = \lambda \hat{b}_t$ *with* $\psi_r > 0$ *and* $\lambda \in (0,1)$.
2. $\frac{\partial \lambda}{\partial \eta} > 0$, $\frac{\partial \lambda}{\partial \delta} > 0$, *and* $\frac{\partial^2 \lambda}{\partial \eta \partial \delta} > 0$.
3. $\frac{\partial \psi_r}{\partial \eta} > 0$; *there exists* $\bar{\kappa} > 0$ *such that if* $\kappa < \bar{\kappa}$, $\frac{\partial \psi_r}{\partial \delta} > 0$ *and* $\frac{\partial^2 \psi_r}{\partial \eta \partial \delta} > 0$.

- Debt-inflation spiral destabilizes the economy $\Rightarrow$ no bounded equilibrium for high δ
- Non-Ricardian behavior amplifies debt-inflation spiral: $\frac{\partial^2 \lambda}{\partial \eta \partial \delta} > 0$
- When the Phillips curve is flat $\kappa \approx 0$, further direct amplification on r^*: $\frac{\partial^2 \psi_r}{\partial \eta \partial \delta} > 0$

Remark. Effect of FD risk on r^* is *always* amplified in medium-run: $\frac{\partial^2 r_t^*}{\partial \eta \partial \delta} > 0$ for large t Detail

Deficit shock and r^*



Taking stock

- Fiscal dominance risk links inflation expectation to government debt.
- Monetary reaction to inflation generates a debt-inflation spiral.
  - through higher interest costs
- Non-Ricardian behaviors amplify the spiral through aggregate demand effect.
- Persistently elevated gov't debt substantially raises r^* in non-Ricardian economies.
  - upward sloping aggregate saving supply

HANK with Fiscal Dominance Risk

Overview

- One-asset HANK model à la Auclert et al. (2025)
  - Uninsurable income risk + borrowing constraint $\rightsquigarrow$ non-Ricardian households
  - Flexible price; sticky wage via union $\rightsquigarrow$ NKPC
  - Labour income tax
  - Short-term government debt (for now)

- Incorporate fiscal dominance risk as before. Solve the model *non-linearly*.
  - Iterating on equilibrium paths in the sequence space (Lin and Peruffo, 2024)

- Probability of fiscal dominance $\delta = 1\%$ quarterly
  - In the ballpark of the DSGE estimates in Bianchi and Melosi (2017)

Calibration

Household

$$V_t(a,z) = \max_{c,a'} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - \varphi \frac{h_t^{1+\phi}}{1+\phi} + \beta \mathbb{E}_t \left[V_{t+1}(a',z') | z \right] \right\}$$

$$c + a' = (1-\tau_t)(w_t h_t z)^{1-\xi} + \mathcal{T}_t + (1+r_t)a$$

$$\log z' = \mu_z + \rho_z \log z + \sigma_z \epsilon_z', \quad \epsilon_z' \sim N(0,1)$$

$$a' \geq \underline{a}$$

- Save in risk-free government bond; hours chosen by unions to satisfy demand
- Labor tax rate τ_t adjusts to implement fiscal rule
- $\mathcal{T}_t$ is lump-sum transfer from the government

Union and Firm

- Unions set wages to maximize the utility of a stand-in hh subject to adj. cost:

$$\log \Pi_t^w = \xi^w \left(\varphi h_t^{1+\phi} - C_{it}^{-\sigma} w_t h_t \right) + \beta \mathbb{E}_t \log \Pi_{t+1}^w$$

  Hours are allocated uniformly $h_{it} = h_t$.

- Representative firm produces output using linear technology:

$$Y_t = \int z_{it} h_{it} \, di = h_t$$

- Profit maximization leads to $w_t = 1$ and $\Pi_t^w = \Pi_t$.

Government

- Government intertemporal budget constraint:

$$B_{t+1} = R_t^n \left(\frac{B_t}{\Pi_t} + G_t + \mathcal{T}_t - T_t \right)$$

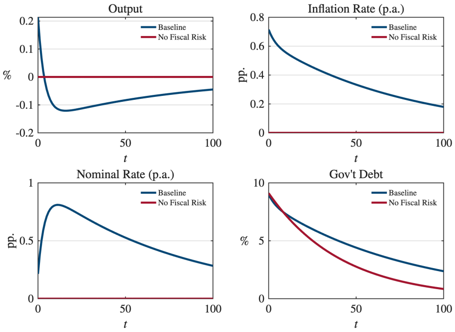
- Monetary and fiscal policy:

$$\frac{R_t^n}{R_{ss}} = \left(\frac{R_{t-1}^n}{R_{ss}} \right)^{\rho_R} \left(\frac{\Pi_t}{\Pi_{ss}} \right)^{(1-\rho_R)\phi_\pi}$$

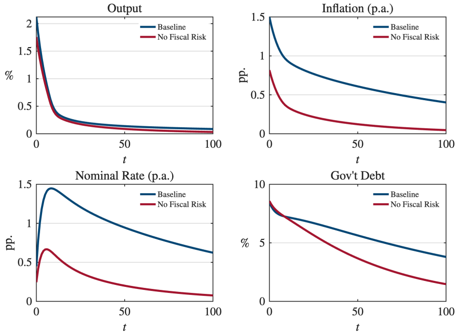
$$T_t = T_{ss} + (1 - \rho_B) \left(\frac{B_t}{\Pi_t} - B_{ss} \right)$$

  - **ML regime:** inertial Taylor rule ($\rho_r = 0.8, \phi_\pi = 1.5$); tax responds to debt ($\rho_b = 0.974$)
  - **FD regime:** constant nominal rate ($\rho_r = \phi_\pi = 0$); constant tax revenue ($\rho_b = 1$)

Deficit-financed Transfer – RANK vs. HANK



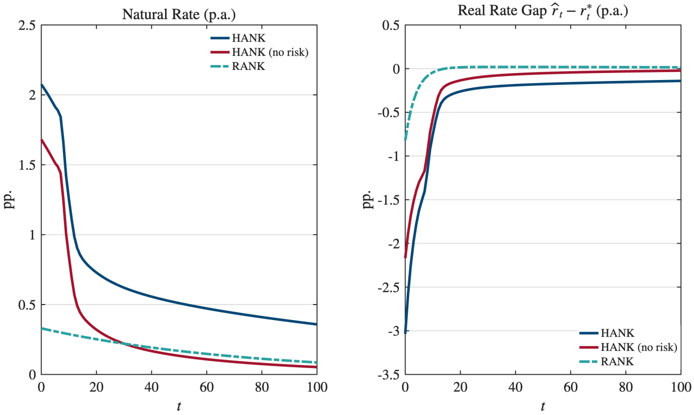
(a) RANK



(b) HANK

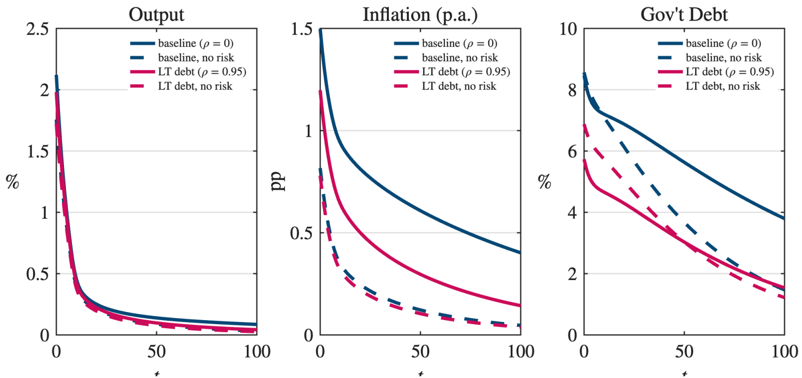
- In HANK, FD risk $\rightsquigarrow$ slightly *larger* y + significantly higher π
  - because of endogenously higher r^*

Fiscal Dominance Risk and r^* in HANK



- Persistent increase in r^* and nonlinearity, consistent with theory
- FD risk $\rightsquigarrow$ higher for longer r^* $\rightsquigarrow$ MP too accommodative $\rightsquigarrow$ higher output

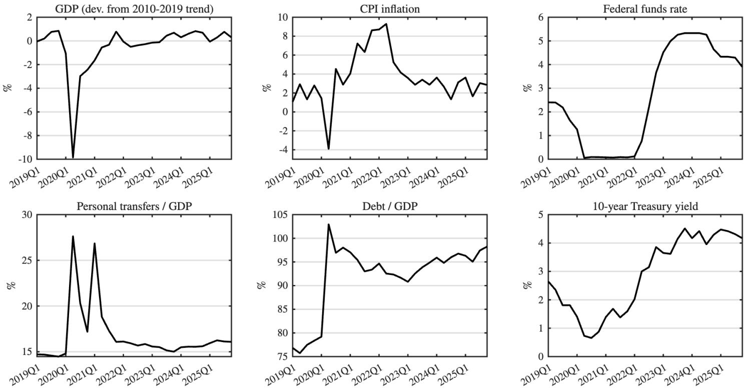
Long-term Debt



- Geometric maturity structure (cf. Cochrane 2001): coupon stream $1, \rho, \rho^2, \ldots$
- Monetary tightening lowers the bond price, dampening the effects of FD risk
- Set $\rho = 0.95$ (i.e., avg. maturity $=$ 5 years) as the baseline

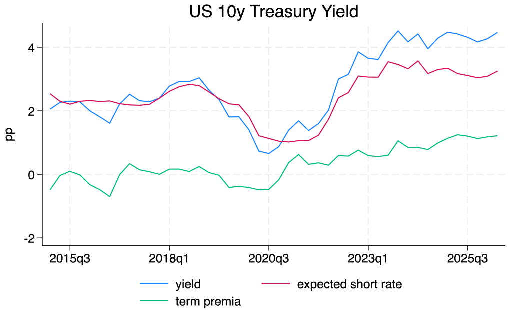
Post-pandemic Rise in Long-term Interest Rates

U.S. 2019-2025

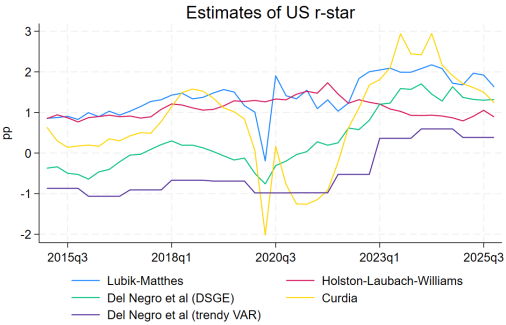


- Severe economic downturn in 2020, aggressive monetary-fiscal support.
- Fiscal transfer total 20% of 2019 GDP.
- 10-year treasury yield 1.9pp higher post pandemic; inflation remains above Fed's target.

Long-term Interest Rates



Source: San Francisco Fed



Post 2023 vs. 2015-2019:

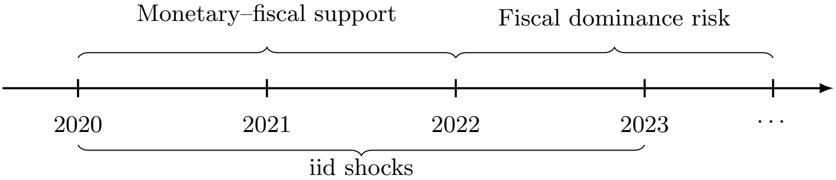
- 10y yield rises by 1.9pp: 0.9pp from expected short rate + 1pp from term premia
- r^* rises by about 1pp

Simulation exercise

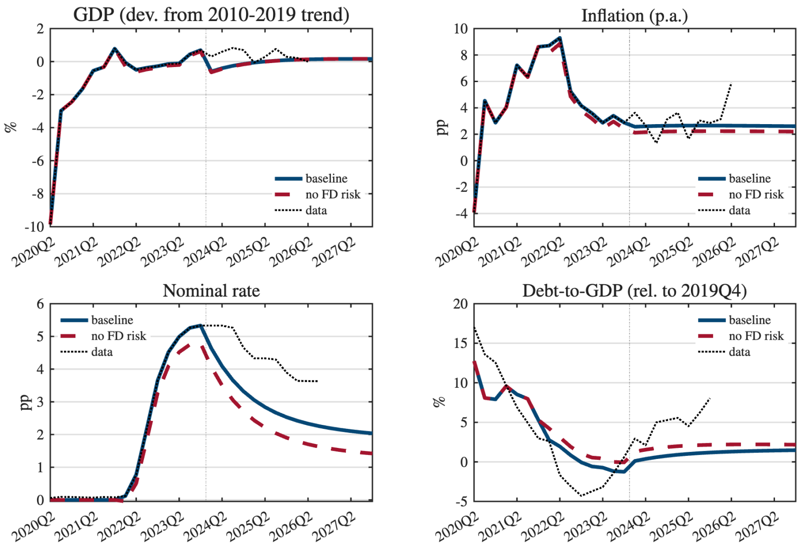
- Feed into the model monetary and fiscal policy during 2020-2021.
    - Fiscal transfer: CARES (2020Q2), CAA (2020Q4), ARPA (2021Q2)
    - Forward guidance: expected ZLB duration from SPF

- Fiscal dominance risk rises permanently from 0 to 1% in 2022Q1.
    - Consistent with inflation option data (Hilscher et al., 2025)

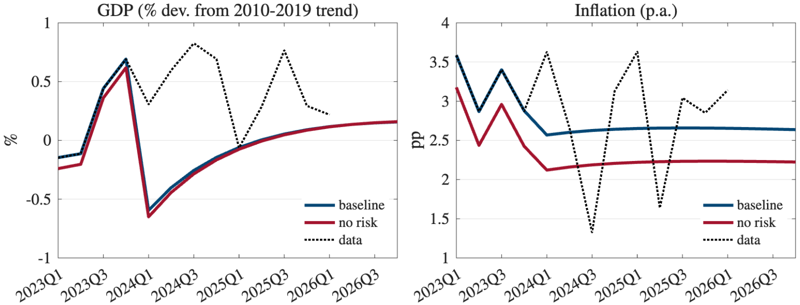
- Calibrate a sequence of shocks to match GDP, inflation, and FFR during 2020-2023.
    - discount factor, cost-push, and monetary policy shock
    - *iid* shock $\rightsquigarrow$ no ad-hoc persistent dynamics



Model Fit

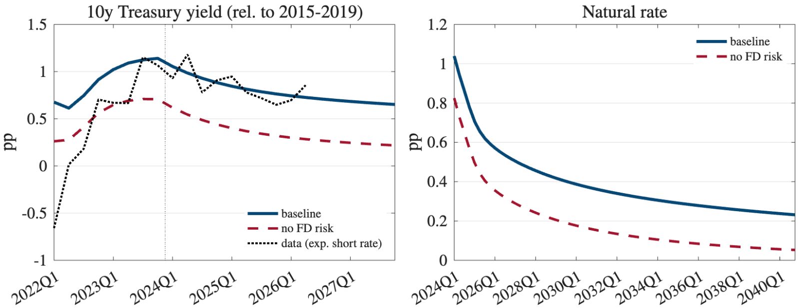

GDP (dev. from 2010-2019 trend)
Inflation (p.a.)
Nominal rate
Debt-to-GDP (rel. to 2019Q4)
%
pp
baseline
no FD risk
data
2020Q2
2021Q2
2022Q2
2023Q2
2024Q2
2025Q2
2026Q2
2027Q2

GDP and Inflation Dynamics after 2023



- **No risk counterfactual.** shut down FD risk, holding shocks fixed.
- FD risks has little effects on GDP
- But inflation is substantially more persistent
  - avg after 2023: 2.74% in the data, 2.6% in the model

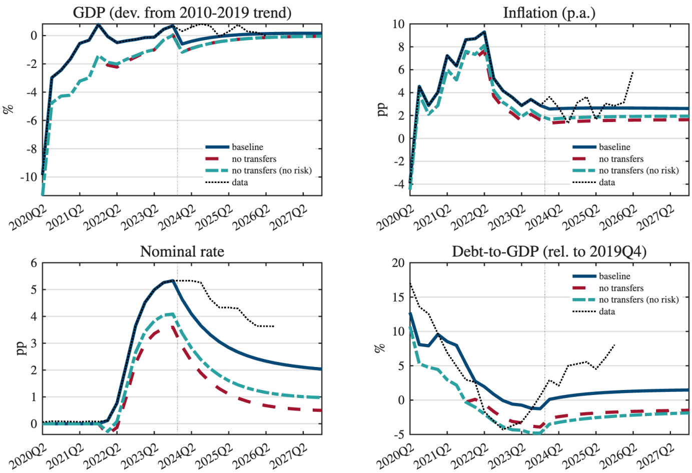
Long-term Interest Rate Dynamics after 2023



- 10y yield in the model consistent with expected-short-rate component in the data
- Significantly higher r^* due to FD risk
  - avg 0.3pp higher during 2030-2040: 0.2pp attributed to FD risk

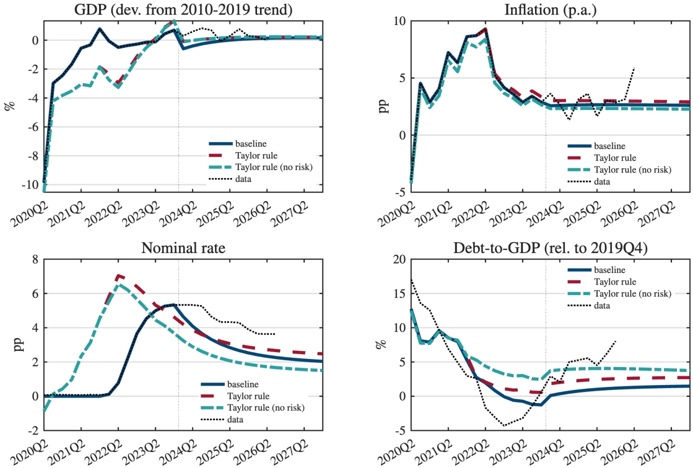
RANK

Counterfactual: No Fiscal Transfer



- More severe recession, but no sticky inflation

Counterfactual: Earlier Monetary Tightening



- **Counterfactual.** MP always follows Taylor rule, no MP shocks
- With FD risk, earlier tightening makes inflation higher and more persistent, and output lower

r^*

Optimal Monetary Policy

Overview

- How should monetary policy react to fiscal stimulus under fiscal dominance risk?
  - **Challenge.** Monetary tightening creates its own inflation

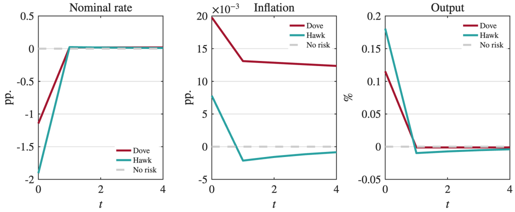
- Solve for optimal policy under commitment, taking into account FD risk
  - Full commitment in the monetary-led regime

- **Approach.** LQ solution in the sequence space
  - Standard dual-mandate objective to focus on output-inflation tradeoff

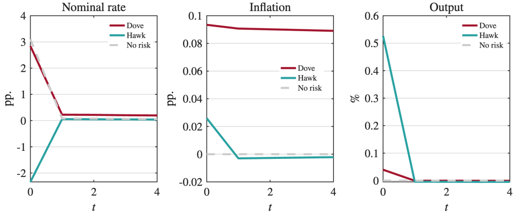
$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 (\alpha_y \hat{y}_t^2 + \alpha_\pi \hat{\pi}_t^2)$$

  - Extend McKay and Wolf (2023) Detail

Optimal Monetary Policy in HANK and RANK



(a) RANK



(b) HANK

Shock: one-time transfer of 1% of GDP

- Dove: $\alpha_y = 1, \alpha_\pi = 0$
- Hawk: $\alpha_y = 0, \alpha_\pi = 1$

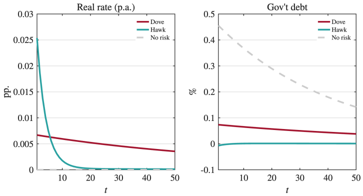
In RANK:

- Optimal MP always reduces r^n on impact.
- Finance deficit to prevent debt-inflation spiral.

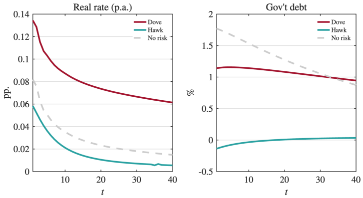
In HANK:

- Only reduce r^n on impact if hawkish.
- Direct stimulus effect prevents rate cuts.

Optimal Monetary Policy in HANK and RANK



(a) RANK



(b) HANK

Figure: real rate and gov't debt *after first period*

- Dove: $\alpha_y = 1, \alpha_\pi = 0$
- Hawk: $\alpha_y = 0, \alpha_\pi = 1$

In RANK:

- Real rate is close to the SS level b/c r^* barely moves

$\rightsquigarrow$ Optimal MP = monetary financing of deficits

In HANK:

- Real rate significantly above the SS level b/c r^* increases

$\rightsquigarrow$ Optimal MP in general:

1. Accommodate the stimulus in the short run
2. Set a higher real rate in the medium to long run

Conclusion

Conclusion

- Fiscal dominance risk constrains monetary policy through a debt-inflation spiral mechanism
  - Higher interest rate $\rightsquigarrow$ increased debt burden $\rightsquigarrow$ rising inflation expectation
- In non-Ricardian economies, r^* persistently higher after fiscal stimulus
  - **Mechanism.** Inelastic saving supply interacts with debt-inflation spiral
  - In a quantitative model, Covid-era stimulus accounts for 30% of the post-pandemic rise in r^*
- Optimal MP in general accommodates fiscal stimulus in the short run, regardless of heterogeneity
  - Key is to prevent the debt-inflation spiral by financing the deficits early on
  - Policy diverges over longer horizons in HANK relative to RANK due to movement in r^*

Reference

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Calibration

Parameter	Interpretation	Value	Source/Target
Households			
σ	CRRA coeff.	2	Standard
ϕ	Inverse Frisch elas.	1	Standard
r	Real rate (p.a.)	0.01	Standard
$\underline{a}$	Borrowing limit	0	Standard
β	Discount factor	0.98	$B_{ss}/Y_{ss} = 2.72$
ρ_z	Autocorr. z	0.98	Storesletten et al. (2004)
σ_z	Std. z	0.12	Storesletten et al. (2004)
Nominal Rigidities			
κ	Slope of NKPC	0.0138	Hazell et al. (2022)
Monetary and Fiscal Policy			
ρ_r	Taylor rule (inertia)	0.80	Baseline
ϕ_π	Taylor rule (inflation)	1.5	Baseline
ρ_b	Fiscal rule	0.974	Angeletos et al. (2024)
ξ	Labor tax progressivity	0.181	Heathcote et al. (2017)
G_{ss}/Y_{ss}	Gov't spending	0.20	Standard
B_{ss}/Y_{ss}	Gov't debt	2.72	Hall and Sargent (2025)
fiscal dominance risk			
δ	Prob. of fiscal dominance	0.01	Baseline

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Fiscal Dominance Risk and r^* in Medium-run

Proposition

Suppose $\alpha, \eta \in (0,1)$ *satisfy* $(1-\alpha)\eta < 1-\rho_B$. *Suppose the economy starts with* $\hat{b}_0 > 0$ *and no further shocks occur. There exist* $\bar{\delta} > 0$ *and* $T \in \mathbb{N}$ *such that if* $\delta \in [0, \bar{\delta})$, *a unique bounded equilibrium exists and for any* $t > T$,

$$\frac{\partial r_t^*}{\partial \delta} > 0 \quad \textit{and} \quad \frac{\partial^2 r_t^*}{\partial \eta \partial \delta} > 0.$$

Intuition. $r_t^* = \psi_r \lambda^t \hat{b}_0 \Rightarrow$ effect on λ dominantes for large t

- Deficit-finance stimulus has persistent effects on r^*

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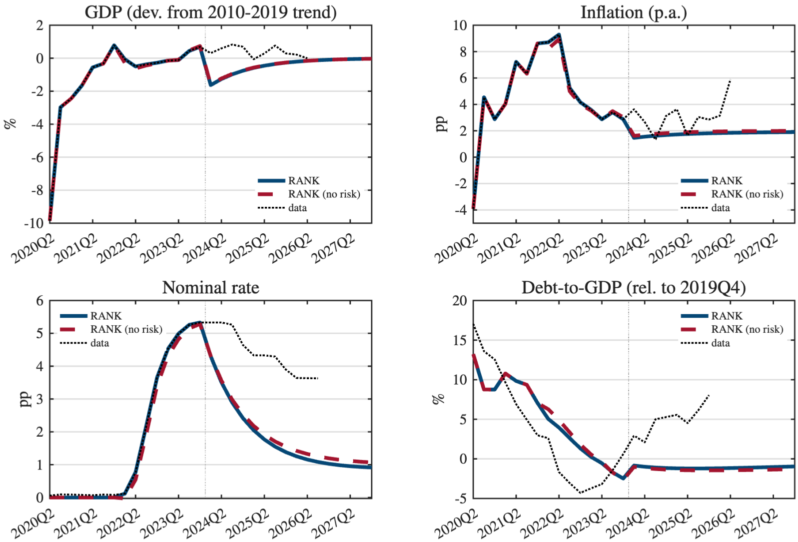
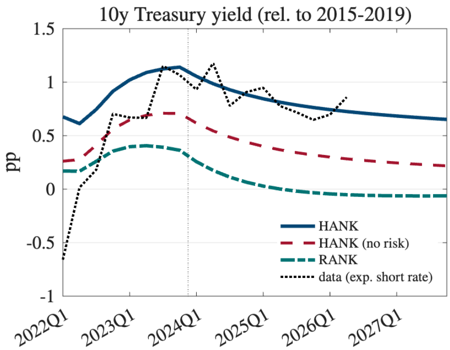
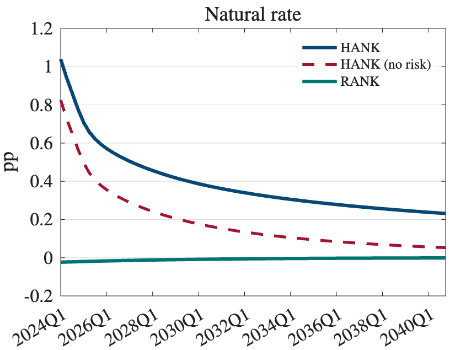
RANK simulation



RANK simulation





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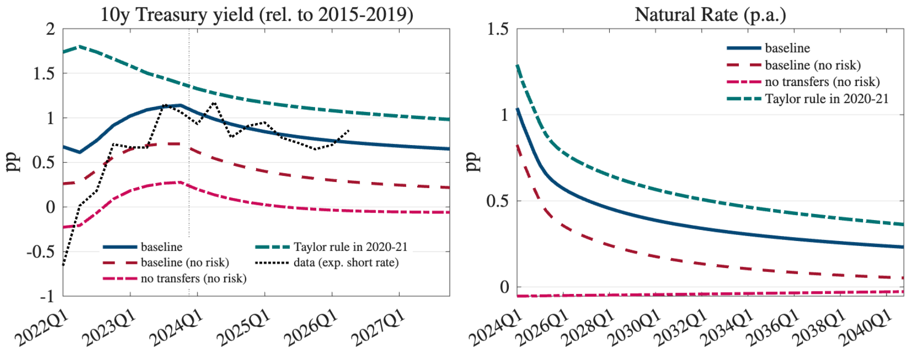
10y yield and r^* across counterfactual



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LQ Problem in the Sequence Space – Objective

- Dual-mandate quadratic loss objective function:

$$\mathcal{L} = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\alpha_y \hat{y}_t^2 + \alpha_\pi \hat{\pi}_t^2\right)$$

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$$= \underbrace{\sum_{t=0}^{\infty} [\beta(1-\delta)]^t \left(\alpha_y \hat{y}_t^2 + \alpha_\pi \hat{\pi}_t^2\right)}_{\text{Monetary-led regime}} + \underbrace{\beta\delta \sum_{\tau=1}^{\infty} [\beta(1-\delta)]^{\tau-1} \left(\sum_{j=0}^{\infty} \beta^j \left(\alpha_y \hat{y}_{\tau,j}^2 + \alpha_\pi \hat{\pi}_{\tau,j}^2\right)\right)}_{\text{Fiscal-dominant regime}}$$

LQ Problem in the Sequence Space – Objective

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- Express the objective in the sequence-space form:

$$\mathcal{L} = \frac{1}{2} \left[\hat{x}' \Omega_0 \hat{x} + \beta\delta \sum_{\tau=1}^{\infty} [\beta(1-\delta)]^{\tau-1} \left(\hat{x}^{\tau\prime} \Omega_1 \hat{x}^{\tau} \right) \right]$$

where $x = (y, \pi)$, $\Omega_0 := \mathrm{diag}(\alpha_y, \alpha_\pi) \otimes \mathrm{diag}(1, \beta(1-\delta), \dots)$,
$\Omega_1 := \mathrm{diag}(\alpha_y, \alpha_\pi) \otimes \mathrm{diag}(1, \beta, \dots)$.

LQ Problem in the Sequence Space – Constraints

- The linearized equilibrium conditions for the monetary-led regime can be written as

$$\mathcal{H}_x \hat{x} + \underbrace{\sum_{\tau=1}^{\infty} \mathcal{H}_x^{\tau} \hat{x}^{\tau}}_{\text{Expectation channel}} + \mathcal{H}_r \hat{r}^n + \mathcal{H}_{\mathcal{T}} \epsilon^{\mathcal{T}} = 0$$

where the $\mathcal{H}$'s are conformable linear operators and $\epsilon^{\mathcal{T}}$ is the fiscal stimulus shock.

LQ Problem in the Sequence Space – Constraints

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  where the $\mathcal{H}$'s are conformable linear operators and $\epsilon^{\mathcal{T}}$ is the fiscal stimulus shock.

- Equivalently, we can express the conditions in terms of impulse responses

$$\hat{x} = \mathcal{G}_r \hat{r}^n + \mathcal{G}_\epsilon \epsilon^{\mathcal{T}}$$
$$\hat{x}^{\tau} = \mathcal{G}_r^{\tau} \hat{r}^n + \mathcal{G}_\epsilon^{\tau} \epsilon^{\mathcal{T}} \quad \forall \tau \geq 1$$

  where the $\mathcal{G}$'s are the IRFs.

LQ Problem in the Sequence Space – Solution

- Full LQ problem:

$$\min_{\hat{x},\{\hat{x}^{\tau}\}_{\tau},\hat{r}^{n}} \frac{1}{2}\left[\hat{x}'\Omega_0\hat{x}+\beta\delta\sum_{\tau=1}^{\infty}[\beta(1-\delta)]^{\tau-1}\left(\hat{x}^{\tau'}\Omega_1\hat{x}^{\tau}\right)\right]$$

$$s.t.\quad \hat{x}=\mathcal{G}_r\hat{r}^n+\mathcal{G}_\epsilon\epsilon^{\mathcal{T}}$$

$$\hat{x}^{\tau}=\mathcal{G}_r^{\tau}\hat{r}^n+\mathcal{G}_\epsilon^{\tau}\epsilon^{\mathcal{T}}\quad\forall\tau$$

- Solution:

$$\hat{r}^n=-\left[\mathcal{G}_r'\Omega_0\mathcal{G}_r+\beta\delta\sum_{\tau=1}^{\infty}[\beta(1-\delta)]^{\tau-1}\left(\mathcal{G}_r^{\tau'}\Omega_1\mathcal{G}_r^{\tau}\right)\right]^{\dagger}\left[\mathcal{G}_r'\Omega_0\mathcal{G}_\epsilon+\beta\delta\sum_{\tau=1}^{\infty}[\beta(1-\delta)]^{\tau-1}\left(\mathcal{G}_r^{\tau'}\Omega_1\mathcal{G}_\epsilon^{\tau}\right)\right]\epsilon^{\mathcal{T}}$$

- Note that the formula holds for both HANK and RANK.

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